QERM 598 - HW 1
Due January 16, 2008
Eli Gurarie

Distributions and CLT

1. First, some practise using `pnorm` and `qnorm`: Let $Z \sim N(0,1)$ and $X \sim N(3,2)$ where the second number is the variance, not the standard deviation. Use R to calculate:
   (a) $\Pr(Z < .4)$
   (b) $\Pr(X < 2)$
   (c) $\Pr(-1 < Z < 1)$
   (d) Solve for $a : Pr(Z < a) = .6$
   (e) Solve for $a : Pr(-a < X < a) = .5$

2. QQ-plots: Emulate the `qqnorm` function in `R` using one line of code that combines exactly four of the functions we discussed in the lab. Test it on data generated from several distributions, including the uniform distribution, the cauchy distribution, and the gamma distribution. How would you characterize the deviations of the QQ-plots from the straight line expected for a normal distribution? What mathematical properties of the distribution do different kinds of deviations from linearity correspond to?

3. Consider a random variable X drawn from a gamma distribution, which has the following pdf:
$$X \sim f(x|\alpha,\beta) = x^{\alpha-1}\frac{\beta^\alpha\, e^{-\beta\, x}}{\Gamma(\alpha)} \text{ for } x > 0. \tag{1}$$
   The mean and variance of X are α/β and α/β^2 respectively.
   (a) Methods of moments estimators (MME's) are obtained by equating the theoretical moments of a distribution (mean, variance, etc.) to the empirical moments ($m_1 = \frac{1}{n}\sum_1^n x$, $m_2 = \frac{1}{n}\sum_1^n x^2$, $m_k = \frac{1}{n}\sum_1^n x^k$) and solving for the parameters. What are the MMEs $\hat{\alpha}$ and $\hat{\beta}$ in terms of m_1 and m_2?
   (b) Convince yourself that these are good estimates by generating vectors of length N drawn from gamma distributions with several parameter values for α and β and different lengths (100, 1000, etc.). Plot histograms of these distributions, superimposing curves for the true distribution and the estimated distribution based on the MME's.
   (c) Perform a numerical experiment to see whether the MME's are biased or not.

4. If X_i where $i = 1, 2, 3, ..., n$ is a vector of iid random variables with mean μ, show graphically using simulations that the variance on the estimate of the mean $\bar{X} = \hat{\mu}$ is inversely proportional to n. Do this for several distributions of your choice.